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RECENT RESULTS ON QCD CORRECTIONS TO SEMILEPTONIC b-DECAYS*

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ABSTRACT

We summarize recent results on QCD corrections to various observables in semileptonic b quark decays. For massless leptons in the final state we present effects of such corrections on triple differential distribution of leptons which are important in studies of polarized b quark decays. Analogous formulas for distributions of neutrinos are applicable in decays of polarized c quarks. In the case of decays with a τ lepton in the final state mass effect of τ has to be included. In this case we concentrate on corrections to the total decay width.

1. Introduction

Studies of the b are one of the most promising directions in the high energy physics in the '90s[1,2]. Wealth of recent results from experiments at LEP and CESR, together with older data from ARGUS at DESY, already determine some properties of the b sector very well. In the relatively near future new experiments HERA-B at DESY, LHC-B at CERN, and B-factories at SLAC and at KEK, will be able to study b physics with high precision. Therefore it is timely to prepare precise theoretical predictions for the properties of decays of the b quark.

In this paper we summarize our recent results concerning QCD corrections in semileptonic b decays. Section 2 deals with energy-angular distributions of massless leptons and section 3 with the total width of the decay $b \to c\tau\bar{\nu}_\tau$.

2. Polarized bottom and charm quarks

Polarization studies for heavy flavors at LEP are a new interesting field of potentially fundamental significance, see Refs.3 and 4 for recent reviews. According to the Standard Model $Z^0 \to b\bar{b}$ and $Z^0 \to c\bar{c}$ decays can be viewed as sources of highly polarized heavy quarks. The degree of longitudinal polarization is fairly large, amounting to $\langle P_b \rangle = -0.94$ for b and $\langle P_c \rangle = -0.68$ for c quarks[5]. The polarizations depend weakly on the production angle. QCD corrections to Born result are about 3%[6]. The real drawback is that due to hadronization the net longitudinal polarization

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of the decaying b and c quarks is drastically decreased. In particular these b quarks become depolarized which are bound in B mesons both produced directly and from $B^* \to B\gamma$ transitions. The signal is therefore significantly reduced. Only those b's (a few percent) which fragment directly into Λ_b baryons retain information on the original polarization[7]. Polarization transfer from a heavy quark Q to the corresponding Λ_Q baryon is 100% [8] at least in the limit $m_Q \to \infty$. Thus, a large net polarization is expected for heavy quarks in samples enriched with these heavy baryons.

It has been proposed long ago[9] that distributions of charged leptons from semileptonic decays of beautiful hadrons can be used in polarization studies for b quarks. Some advantages of neutrino distributions have been also pointed out[10,11,12]. Recently there has been considerable progress in the theory of the inclusive semileptonic decays of heavy flavor hadrons. It has been shown that in the leading order of an expansion in inverse powers of heavy quark mass $1/m_Q$ the spectra for hadrons coincide with those for the decays of free heavy quarks[13] and there are no Λ_{QCD}/m_Q corrections to this result away from the energy endpoint. Λ^2_{QCD}/m^2_Q corrections have been calculated in Refs.14,15 for B mesons and in ref.15 for polarized Λ_b baryons. For some decays the results are similar to those of the well-known $ACCMM$ model[16]. The corrections to charm decays are larger than for bottom and convergence of $1/m_Q$ expansion is poorer[17]. Perturbative first order QCD corrections contribute 10-20% to the semileptonic decays and for bottom are larger than the nonperturbative ones.

In our recent article[18] compact analytic formulae have been obtained for the distributions of the charged lepton and the neutrino. These formulae agree with the results of our earlier calculations[19,10] for the joint angular and energy distribution of the charged lepton in top, charm and bottom quark decays. The QCD corrected triple differential distribution of the charged lepton for the semileptonic decay of the polarized quark with the weak isospin $I_3 = \pm 1/2$ can be written in the following way[18]:

$$\frac{\mathrm{d}\Gamma^\pm}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\cos\theta} \sim \Big[\mathrm{F}_0^\pm(x,y) + S\cos\theta\,\mathrm{J}_0^\pm(x,y)\Big] - \frac{2\alpha_s}{3\pi}\Big[\mathrm{F}_1^\pm(x,y) + S\cos\theta\,\mathrm{J}_1^\pm(x,y)\Big] \tag{1}$$

In the rest frame of the decaying heavy quark θ denotes the angle between the polarization vector $\vec{s}$ of the heavy quark and the direction of the charged lepton, $S = |\,\vec{s}\,|$, $x = 2Q\ell/Q^2$ and $y = 2\ell\nu/Q^2$ where Q, ℓ and ν denote the four-momenta of the decaying quark, charged lepton and neutrino. Eq.(1) describes also the triple differential distribution of the neutrino for $I_3 = \mp 1/2$. In this case, however, $x = 2Q\nu/Q^2$ and θ denotes the angle between $\vec{s}$ and the three-momentum of the neutrino. The functions $\mathrm{F}_0^\pm(x,y)$ and $\mathrm{J}_0^\pm(x,y)$ corresponding to Born approximation read:

$$\mathrm{F}_0^+(x,y) = x(x_m - x) \tag{2}$$

$$\mathrm{J}_0^+(x,y) = \mathrm{F}_0^+(x,y) \tag{3}$$

$$\mathrm{F}_0^-(x,y) \;=\; (x-y)(x_m - x + y) \tag{4}$$

$$\mathrm{J}_0^-(x,y) \;=\; (x-y)(x_m - x + y - 2y/x) \tag{5}$$

where $x_m = 1-\epsilon^2$, $\epsilon^2 = q^2/Q^2$ and q denotes the four-momentum of the quark originating from the decay. The functions $\mathrm{F}_1^\pm(x,y)$ and $\mathrm{J}_1^+(x,y)$ correspond to the first order QCD corrections and are given in Ref.18.

Non-trivial cross checks are fulfilled by the polarization independent parts of the distributions (1):

- the distributions $\mathrm{d}\Gamma^\pm/\mathrm{d}x\,\mathrm{d}y$ agree with the results for unpolarized decays which were obtained by Jeżabek and Kühn[20]. The present formulae are simpler.

- in the four-fermion (Fermi) limit integration over y can be performed numerically. The resulting distributions $\mathrm{d}\Gamma^\pm/\mathrm{d}x$ also agree with those of Ref.20. Recently the results of Ref.20 have been confirmed[21]. Thus an old conflict with other calculations[22] is solved and the agreement with Ref.20 can be considered as a non-trivial cross check. Moreover, the analytic result[20] for $\mathrm{d}\Gamma^+/\mathrm{d}x$ and $\epsilon = 0$ has been also confirmed[23].

- $$\mathrm{d}\Gamma^+/\mathrm{d}y = \mathrm{d}\Gamma^-/\mathrm{d}y$$

  and the analytic formula for this distribution exists[24] which at the same time describes the lifetime of the top quark as a function of its mass. This formula has been confirmed by a few groups, c.f. Ref.25 and references therein.

- in the four-fermion limit the result for the total rate Γ derived from eq.(1) agrees with the results of Ref.26 and the analytical formula of Ref.27.

3. QCD corrections to $b \to c\tau\bar{\nu}_\tau$

The semileptonic decay of b quark in the case of massive lepton in the final state is a particularly interesting process. In contrast to the case of light leptons the decay rate is sensitive to the coupling of the scalar component of the W boson. It can also be strongly affected by the charged Higgs boson predicted by many extensions of the standard model, e.g. by the minimal supersymmetric standard model. Recently the branching ratio has been measured: $\mathrm{Br}(b \to \tau\nu X) = (2.75 \pm 0.3 \pm 0.3)\%$[1].

The perturbative QCD corrections to the inclusive decay rate have been evaluated numerically[28]. The bound stated corrections have also been computed using Heavy Quark Effective Theory[28,29,30]. Recently we have presented the analytical formula for the QCD corrections in the form of a one dimensional intergral over the invariant mass squared w^2 of the leptons[31]:

$$\Gamma(b \to c\tau\bar{\nu}_\tau) \;=\; \int_\eta^{(1-\sqrt{\rho})^2} \frac{\mathrm{d}\Gamma}{\mathrm{d}t}\,\mathrm{d}t \tag{6}$$

with the differential rate

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t} = \Gamma_{bc}\left(1-\frac{\eta}{t}\right)^2\left\{\left(1+\frac{\eta}{2t}\right)\left[\mathcal{F}_0(t)-\frac{2\alpha_s}{3\pi}\mathcal{F}_1(t)\right]+\frac{3\eta}{2t}\left[\mathcal{F}_0^s(t)-\frac{2\alpha_s}{3\pi}\mathcal{F}_1^s(t)\right]\right\} \quad (7)$$

where

$$\Gamma_{bc} = \frac{G_F^2 m_b^5|V_{cb}|^2}{192\pi^3} \quad (8)$$

and we have used the following dimensionless variables

$$\rho = m_c^2/m_b^2 \qquad \eta = m_\tau^2/m_b^2 \qquad \text{and} \qquad t = \mathrm{w}^2/m_b^2$$

and the explicit formulas for the functions $\mathcal{F}_i$ are given, together with a detailed derivation, in our paper[31]. The integrated decay rate can be rewriten as

$$\Gamma(b\to c\tau\bar{\nu}_\tau) \;=\; \Gamma^{(0)}\left[1-\frac{2\alpha_s}{3\pi}F(m_b,m_c,m_\tau)\right] \quad (9)$$

In Fig. 1 we present the dependence of the relative correction $F(m_b, m_c, m_\tau)$ on m_b for two different values of $m_b - m_c$. We also plot an analogous function $F(m_b, m_c, 0)$ relevant for the decays with electrons or muons in the final state. One can see that the relative corrections are smaller for a heavier lepton and for a heavier quark in the final state.

An important feature of formula 7 is that it gives the QCD correction to the decay rate differentiated with respect to the square of the invariant mass of the lepton system. The perturbation theory is more reliable in the region of smaller w^2, so it is useful to have the correction given as a function of w^2. We rewrite eq. 7 as

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t} = \frac{\mathrm{d}\Gamma^{(0)}}{\mathrm{d}t}\left[1-\frac{2\alpha_s}{3\pi}G(t)\right]. \quad (10)$$

The shape of the normalized Born distribution is shown in Fig. 2a. The function $G(t)$ is plotted in Fig. 2b for τ as well as for massless leptons.

Further theoretical study of the decay $b \to c\tau\bar{\nu}_\tau$ is needed. Especially important is the determination of QCD corrections to the energy spectrum of the τ. This is important for the current measurements of the branching ratio, since these are based on an assumed distribution of the missing energy. It will become even more important in the future measurements of the leptonic decay $B \to \tau\bar{\nu}_\tau$, where τ's from the channel $b \to c\tau\bar{\nu}_\tau$ are the major background. The leptonic decay is going to be the source of determination of the B decay constant f_B, crucial in the efforts to determine the weak mixing angles and overconstrain the unitarity triangle.

The QCD corrections to the semileptonic decay of b into τ are also applicable to the decay $b \to c\bar{c}s$.

$$\Gamma(b\to c\bar{c}s) = \Gamma_{c\bar{c}s}^{(bare)}\left[1+\tfrac{\alpha_s}{\pi}\left(\delta_{bc}+\delta_{\bar{c}s}+\delta_{penguin}\right)\right] \quad (11)$$

where $\Gamma_{c\bar{c}s}^{(bare)}$ is the rate without any QCD corrections, δ_{bc} arises from gluon exchange on the bc line, $\delta_{\bar{c}s}$ is due to gluon interactions within the $\bar{c}s$ loop, and $\delta_{penguin}$ is due to effects of the penguin type; cf. Ref. 32 where $\delta_{\bar{c}s}$ is calculated as an integral over the invariant mass of the $\bar{c}s$ system. Our calculation provides the missing part of the correction δ_{bc} (see also Ref. 33). We plot δ_{bc} in Fig. 3 for four different values of $m_b - m_c$.

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6. List of figures

1. Relative QCD correction to the total decay width as a function of mass of the b quark for $m_b - m_c = 3.414$ GeV and 3.317 GeV: for a massless lepton (solid and dashed lines) and for the τ lepton (dotted and dash-dotted lines).

2. (a) Normalized Born differential decay rates for massless leptons (solid) and for the τ lepton (dashed); (b) QCD correction $G(t)$ to the differential decay width for a massless lepton (solid) and for τ (dotted).

3. QCD correction δ_{bc} for the decay $b \to c\bar{c}s$ as a function of m_b, for various values of $m_b - m_c$: 3.2 GeV (solid), 3.3 GeV (dashed), 3.4 GeV (dotted), and 3.5 GeV (dash-dotted).